PUBLIC POLICY INSTITUTE OF CALIFORNIA

Technical Appendix

# Consumption Tax Options for California

Alan J. Auerbach

## Contents



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# A Simple Two-State Model

This appendix lays out and analyzes a model of the effects of taxation by one state in a two-state economy. The model is very simple and leaves out much of the detail relevant to the implementation of actual tax systems, but it provides many useful insights regarding how different taxes affect the economy and what their incidence and efficiency effects are. At the end of this appendix, we briefly discuss how various extensions of the model might affect the results.

## The Basic Model

Each state is assumed to produce a single, traded good, which can be used for final consumption or as an intermediate input. In addition to the intermediate input, the other factors of production are capital and a fixed local factor, which can be viewed as some combination of labor and land. Capital is fixed nationally, but freely mobile across the two states, and owned by individuals in the two states in proportion to their fixed factor endowments. Trade is costless so, except for taxes, output prices will be equal across jurisdictions. The model of production is symmetric, with the two states differing only in tax policy and relative size. We assume that there is only one production sector for tax purposes, i.e., there are no distinct corporate and noncorporate sectors.

The constant returns to scale production function in each state is $X = f(K, L, M)$, where $K$ is capital, $L$ is labor, and $M$ is intermediate inputs. Output, $X$, is devoted either to final consumption, $C$, or to use as intermediate goods. Uses of output must equal output nationally but not necessarily in each state individually. The capital stock is assumed to be owned equally. Note that by having a one-period model, we will not pick up the intertemporal distortion associated with capital taxation that, as discussed in the text, is an issue more relevant for the design of national tax policy than for that of an individual state.

In the absence of taxation, efficient behavior by producers leads to the following marginal conditions with respect to factor inputs:

(1) $p_i f_1 = r$

(2) $p_i \mathrm{f}_2 = w_i$

(3) $p_i \mathrm{f}_3 = q_i$

where $p$ is the price of output sold to consumers, $q$ is the price of output sold to businesses for intermediate use, $w$ is the wage rate (i.e., the return to the fixed factor), and $r$ is the rate of return to capital. The price subscript, $i$, indicates the location of production, except for the rate of return to capital, which is common to the two states by the assumption of capital mobility. In a no-tax equilibrium, the price of intermediate materials, $q$, equals the output price, and the two prices are also equal across the two states. With taxation in place, we may observe four distinct prices, in which case the relevant prices in the above expressions are those where sales occur. Note that because this is a real model with no nominal magnitudes, the level of prices is irrelevant: Only relative prices are defined, and we are free to choose one good or factor as the numeraire commodity.

In addition to the marginal conditions (1)–(3), the model has various adding-up constraints, including that the total capital used in both states equals the amount of capital in the economy,

(4) $K = K_1 + K_2$

and that total output, $X$, produced in the two states equals total use of output for consumption, $C$, and intermediate inputs, $M$:

(5) $$X_1 + X_2 = C_1 + C_2 + M_1 + M_2$$

## Tax Systems

One can use this model to analyze the effects of several tax systems, in each case starting with how the above marginal production conditions are affected by the particular tax system. In each case, we will assume for simplicity that the tax system is imposed only in state 1 and that state 2 has no taxation.

### Retail sales tax (final sales only)

Here, the price received by any producer for sales in state 1, $p_1$, is marked up by the sales tax rate, so that consumers in state 1 face a price $p_1(1 + \tau_1)$. Since there is no taxation of intermediate inputs, the producer prices of both types of output are the same in state 1, $q_1 = p_1$. Also, since there is no taxation in state 2, the producer prices are equal across states, so $q_1 = p_1 = q_2 = p_2$.

Letting $p_1$ be the numeraire, the introduction of taxation has no effect on output prices or factor prices, and hence on factor allocation, either. In this model, a retail sales tax is just a lump-sum tax, because factor incomes are fixed. The incidence falls entirely on those living in state 1 in proportion to their share of income.

### Tax on intermediate sales only

This version of the sales tax applies only to intermediate sales in state 1, so we have no mark-up at the consumer level but a distortion between the consumer price and the input price in state 1. There is no distortion in state 2, and retail sales prices are equal across the two states. That is, $p_1 = p_2 = q_2$; $q_1 = p_1(1 + \tau_1)$.

This case will be analyzed further below.

### Tax on all sales

This case corresponds to a tax on gross receipts, whereas California's existing RST would fall somewhere in between this case and the first. Here, consumers and producers in state 1 face a price, $p_1(1 + \tau_1)$, with $p_1 = p_2 = q_2$.

There will be no need to analyze this case separately, since it is just a combination of the previous two.

### Value added tax

In this case, the tax base is domestic purchases of output minus domestic purchases of intermediate goods:

$$(p_1C_1 + q_1M_1) - q_1M_1$$

It is equivalent to a retail sales tax on final goods, so there is no need to analyze it separately.

### VAT with formula apportionment by sales (i.e., a BNRT)

Here, the tax base equals $(p_1C_1 + p_2C_2)S$

where $S$ is the sales apportionment factor, equal to the share of total sales made in state 1:

(6) $S = (p_1C_1 + q_1M_1)/(p_1C_1 + q_1M_1 + p_2C_2 + q_2M_2)$

This case will be analyzed further below.

### Capital income tax with formula apportionment

Here, the tax base equals $(p_1C_1 + p_2C_2 - w_1L_1 - w_2L_2)F$, where $F$ is a weighted average of $S$, the sales factor defined in expression (6), and comparable factors for assets and payroll, respectively, $K_1/(K_1 + K_2)$ and $w_1L_1/(w_1L_1 + w_2L_2)$. In the case of apportionment based only on sales, the only distinction from the previous case is that only returns to capital are taxed—returns to the fixed factor are deducted from the tax base. This case differs from a sales-apportioned corporate income tax in that it applies to all businesses, not just corporations.

This case will be analyzed further below.

## Detailed Analysis

Here, we consider the effects of different tax systems in more detail, looking at their general equilibrium effects. To keep distinct the effects in states 1 and 2, we will denote the production function in state 2 as $g(\cdot)$, rather than $f(\cdot)$, even though the underlying production functions are the same. That is,

(7) $X_1 = f(K_1, L_1, M_1)$

(8) $X_2 = g(K_2, L_2, M_2)$

For each tax system, we will analyze the effects of introducing a small tax in state 1, starting from a no-tax equilibrium.

### Tax on intermediate sales only

The first-order conditions for a profit-maximizing representative firm operating in both jurisdictions are:

$pf_1 = r$

$pg_1 = r$

$pf_2 = w_1$

$pg_2 = w_2$

$pf_3 = p(1 + \tau) \Rightarrow f_3 = (1 + \tau)$

$pg_3 = p \Rightarrow g_3 = 1$

where we drop the subscript on the price, $p$, since it is the same across jurisdictions.

Total differentiation of the last two terms yields (since labor is fixed in each state):

(9) $f_{31}dK_1 + f_{33}dM_1 = d\tau$

(10) $g_{31}dK_2 + g_{33}dM_2 = 0$

Since we assume that the production functions are the same, we initially have the same factor proportions, so $X_1 f_{ij} = X_2 g_{ij}$, where $X_i$ is output in state $i$. Thus, premultiplying conditions (9) and (10) by the state's output and adding the conditions, we have:

$X_1(f_{31}dK_1 + f_{33}dM_1) + X_2(g_{32}dK_2 + g_{33}dM_2) = X_1 d\tau$

$\Rightarrow$ (since capital overall is fixed) $X_1 f_{33}(dM_1 + dM_2) = X_1 d\tau$

$$\Rightarrow dM = \frac{d\tau}{f_{33}}$$

That is, since by the standard property of diminishing returns ($f_{33} < 0$), $M$ falls—the overall use of intermediate goods is discouraged. Discouraging the use of $M$ in state 1 leads to a larger net reduction overall, the smaller is $f_{33}$. Since $X_1 f_{ij} = X_2 g_{ij}$, we can see that $f_{33}$ will be smaller the larger is $X_1/X_2$, i.e., the more important state 1 is in the national economy.

Total differentiation of the conditions for the choice of capital leads to:

(11) $\quad f_{11}dK_1 + f_{13}dM_1 = dr$

(12) $\quad g_{12}dK_2 + g_{13}dM_2 = dr$

where we have normalized the retail price, $p$, which is equal in both states, to 1, i.e., we have chosen retail sales as the numeraire commodity.

Once again, premultiplying by output and adding, we get from adding (11) and (12):

$X_1 f_{13}(dM_1 + dM_2) = (X_1 + X_2)dr$

$$\Rightarrow dr = \left(\frac{X_1}{X_1 + X_2}\right)\frac{f_{13}}{f_{33}}d\tau$$

Thus, the effect on $r$ is scaled by the relative size of state 1 and the complementarity of capital and intermediate goods. If they are complementary, then capital use is also discouraged and the lower demand for capital reduces its rate of return, $r$; if they are substitutes, then capital use is encouraged and $r$ rises.

Now, to solve for changes in the individual factors of production, we subtract expression (10) from (9) and (12) from (11), instead of adding, getting:

$X_1(f_{31}dK_1 + f_{33}dM_1) - X_2(g_{32}dK_2 + g_{33}dM_2) = X_1 d\tau$

$\Rightarrow 2X_1 f_{31}dK_1 + x_1 f_{33}(dM_1 - dM_2) = X_1 d\tau$

$X_1(f_{11}dK_1 + f_{13}dM_1) - X_2(g_{12}dK_2 + g_{13}dM_2) = (X_1 - X_2)dr$

$\Rightarrow 2X_1 f_{11}dK_1 + X_1 f_{13}(dM_1 - dM_2) = (X_1 - X_2)dr$

But $dM_1 - dM_2 = 2dM_1 - dM$, so these equations become:

$2X_1 f_{31}dK_1 + X_1 f_{33}(2dM_1 - dM) = X_1 d\tau$

$2X_1 f_{11}dK_1 + X_1 f_{13}(2dM_1 - dM) = (X_1 - X_2)dr$

which we can solve for $K_1$ and $M_1$. Multiplying the second equation by ($f_{33}/f_{13}$) and subtracting it from the first yields, after a few steps, the solutions for changes in capital inputs:

$$dK_1 = -dK_2 = \frac{X_2}{X_1 + X_2}\frac{f_{13}}{f_{31}f_{13} - f_{11}f_{33}}d\tau$$

Using the first-order condition (9) above, we get from this the changes in intermediate inputs:

$$dM_1 = \left(1 - \frac{X_2}{X_1 + X_2}\frac{f_{31}f_{13}}{f_{31}f_{13} - f_{11}f_{33}}\right)\frac{d\tau}{f_{33}}$$

$$dM_2 = \frac{X_2}{X_1 + X_2}\frac{f_{31}f_{13}}{f_{31}f_{13} - f_{11}f_{33}}\frac{d\tau}{f_{33}}$$

Finally, we solve for wage rates in the two states by differentiating the first-order conditions for labor,

(13) $dw_1 = f_{21}dK_1 + f_{23}dM_1$

(14) $dw_2 = g_{21}dK_2 + g_{23}dM_2$

and substituting in our solutions for the other variables. This yields:

$$dw_1 = \left(f_{23} + f_{13}\frac{X_2}{X_1 + X_2}\frac{f_{21}f_{33} - f_{23}f_{31}}{f_{31}f_{13} - f_{11}f_{33}}\right)\frac{d\tau}{f_{33}}$$

$$dw_2 = -\left(f_{13}\frac{X_1}{X_1 + X_2}\frac{f_{21}f_{33} - f_{23}f_{31}}{f_{31}f_{13} - f_{11}f_{33}}\right)\frac{d\tau}{f_{33}}$$

Note also that by combining the two conditions we have:

$$dw_2 = \frac{X_1}{X_2}\left(\frac{f_{23}}{f_{33}}d\tau - dw_1\right)$$

### Evaluating effects

First, note that $f_{11}f_{33} - f_{31}f_{13}$ is the determinant of the Hessian of the production function holding labor constant. Since there are decreasing returns to subsets of inputs, this determinant is negative, so $dM_2 > 0$. That is, intermediate inputs fall overall but increase in state 2. Second, capital is helped or hurt depending on complementarity. Third, the effect on labor in state 1 depends on a direct effect, based on complementarity with respect to the intermediate input, plus an indirect effect, via the effect on capital (which also affects the returns to labor in state 2). If all factors are complements, then both terms hurt $w_1$, whereas the shift of intermediate goods use to state 2 helps labor there to offset the effect via capital; in that case, the net effect on labor in state 2 would be positive.

## VAT with formula apportionment by sales (i.e., a BNRT)

Again, we start by thinking of a representative firm. After going through this analysis, we will consider whether incentives might lead to a different outcome in equilibrium.

The representative firm's objective is to maximize profits, which (using expression (6) for the apportionment factor):

$$p_1C_1 + p_2C_2 - \tau(p_1C_1 + p_2C_2)\left(\frac{p_1C_1 + q_1M_1}{p_1C_1 + q_1M_1 + p_2C_2 + q_2M_2}\right) - r(K_1 + K_2) - w_1L_1 - w_2L_2$$

Since the tax is based on the location of sales, there is no tying of sales to the location of production; so the prices, $p$ and $q$, do not depend on whether items are imported or domestic. Also, for a price-taking firm, sales have the same effect on the tax base, regardless of use, so there is no reason for firms to charge a different price for intermediate and final sales in a given jurisdiction (i.e., $p_i = q_i$). This makes the firm's objective:

$$(p_1C_1 + p_2C_2)\left(1 - \tau\frac{p_1(C_1 + M_1)}{p_1(C_1 + M_1) + p_2(C_2 + M_2)}\right) - r(K_1 + K_2) - w_1L_1 - w_2L_2$$

Using expression (5) relating production to purchases of output, we substitute for $C_2$ in this expression to get:

$$\left(p_1C_1 + p_2\left[X_1 + X_2 - C_1 - M_1 - M_2\right]\right)\left(1 - \tau\frac{p_1(C_1 + M_1)}{p_1(C_1 + M_1) + p_2(X_1 + X_2 - C_1 - M_1)}\right) - r(K_1 + K_2) - w_1L_1 - w_2L_2$$

We will take first-order conditions with respect to labor, capital, and intermediate goods in both jurisdictions, as well as final sales in state 1, using equations (7) and (8) to express $X_1$ and $X_2$ in terms of the factors of production used in the respective states.

The first-order condition with respect to final sales in state 1, $C_1$, reduces to:

$$p_1 = \frac{p_2\left(1-(1-R)\tau S\right)}{1-R\tau-(1-R)\tau S}, \tag{15}$$

where

$$R = \frac{p_1 C_1 + p_2 C_2}{p_1\left(C_1+M_1\right)+p_2\left(C_2+M_2\right)} \tag{16}$$

is the share of final sales among all sales, and $S$, as defined in expression (6), is the share of total sales in state 1. Note that if $R = 1$—no intermediate sales—this would be just a sales tax at rate $\tau$ in state 1, its output price increased by that factor relative to the state 2 price.

The first-order conditions for capital in the two states are ($K_1$; same for $K_2$ except substitute $g$ for $f$):

$p_2 f_1(1 - \tau S) + R\tau S p_2 f_1 = r$

$\Rightarrow p_2 f_1(1 - (1 - R)\tau S) = r.$

For labor, the first-order conditions are ($L_1$; same for $L_2$ except substitute $g$ for $f$):

$p_2 f_2(1 - (1 - R)\tau S) = w_1.$

Note also that when $R < 1$—there are intermediate goods used in production—there is effectively a tax on labor and capital income in both states at rate $(1 - R)\tau S$ or, equivalently, if the returns to capital and labor are held constant, an increase in the price of retail sales in the untaxed state 2.

Given this result, the effects of taxation on the price of output in state 1, as described in expression (15), may be seen as the combined effect of an effective increase in general prices in both states (holding returns to labor and capital fixed), by a factor, $(1 - R)\tau S$, plus an additional piece that hits only state 1, $R\tau$, raising the consumer price in state 1 above that in state 2. We can interpret the first piece as relating to increased costs in the use of intermediate inputs, which are included in the apportionment formula. This piece depends on the share of sales that are intermediate, $(1 - R)$, and the share of sales attributed to state 1, $S$. The second piece relates to the tax on final consumption, which hits consumption only in state 1. The weights on these two components depend on the relative importance of intermediate purchases and final purchases. Also, as the relative size of state 1 changes, the size of the nationwide piece changes but not when normalized by the size of the tax. That is, $(1 - R)\tau S$ gets smaller as $S$ declines, but it is applied to a larger economy relative to that of state 1. The incidence calculation for each of these pieces is straightforward, since there would be no behavioral responses to a final sales tax in one or both states, and the final sales tax in a particular state falls on consumers there in proportion to their consumption. Note, in particular, that part of the tax burden is borne by consumers in state 2.

The final effect of this tax system is through the tax wedge placed on intermediate inputs, for which the first-order conditions are:

$M_1$: $f_3 = \dfrac{1-\tau S}{1-R\tau-(1-R)\tau S} = 1+\dfrac{R\tau(1-S)}{1-R\tau-(1-R)\tau S}$

$M_2$: $g_3 = \dfrac{1-\tau S}{1-(1-R)\tau S} = 1-\dfrac{R\tau S}{1-(1-R)\tau S}$

The conditions for M say that there is a tax on intermediate inputs in state 1, but a subsidy in state 2. The firm will be encouraged to sell more intermediate goods in state 2 to reduce its tax liability in state 1, and this effectively places a tax on sales of intermediate goods in state 1 while subsidizing those used in state 2. The effects of these two pieces follow that for intermediate sales taxation considered above, except that in this case the tax in state 1 is accompanied by a subsidy in state 2, so that overall use of intermediate inputs is not discouraged. Note that as the share of state 1 in the economy rises ($S$ increases), the effective tax on intermediate inputs in state 1 decreases. The ability to shift intermediate production to the other state is a necessary part of this distortion. This effect is different from the case of the tax on intermediate sales analyzed above, which suggests that the distortions under the two systems in the choice of intermediate inputs will diverge as the relative size of the state imposing the tax grows.

Letting $p_2$ again be the numeraire, we can follow the same steps as those used for the intermediate sales tax to determine the comparative statics resulting from a change in $\tau$. The first important result is that $dM = 0$. The tax on intermediate inputs in state 1 and the subsidy in state 2 just offset in their effects. As a consequence, the change in $r$ comes only from the general tax component, so

$$dr = -r(1 - R)Sd\tau$$

A related result is that the net incidence on labor is also just due to this general term, but there is also a large distributional effect between state 1 and state 2, because of the shift of $K$ and $M$, so

$$dw_1 = -w(1-R)Sd\tau + \left( f_{23} + f_{13} \frac{x_2}{x_1 + x_2} \frac{f_{21}f_{33} - f_{23}f_{31}}{f_{31}f_{13} - f_{11}f_{33}} \right) \frac{dT_1}{f_{33}} - \left( f_{13} \frac{x_2}{x_1 + x_2} \frac{f_{21}f_{33} - f_{23}f_{31}}{f_{31}f_{13} - f_{11}f_{33}} \right) \frac{dT_2}{f_{33}}$$

$$dw_2 = -w(1-R)Sd\tau + \left( f_{23} + f_{13} \frac{x_1}{x_1 + x_2} \frac{f_{21}f_{33} - f_{23}f_{31}}{f_{31}f_{13} - f_{11}f_{33}} \right) \frac{dT_2}{f_{33}} - \left( f_{13} \frac{x_1}{x_1 + x_2} \frac{f_{21}f_{33} - f_{23}f_{31}}{f_{31}f_{13} - f_{11}f_{33}} \right) \frac{dT_1}{f_{33}}$$

where $dT_1 = R(1 - S)d\tau$ and $dT_2 = -RSd\tau$ are the changes in intermediate taxation (a subsidy in state 2) implicit in the tax change, $d\tau$. Finally, we note that the state 1–specific increase in the retail goods price is

$$dp_1 = Rd\tau$$

Now, let us return to the issue of whether identical firms will be the outcome in equilibrium. We can see that specialization will be preferred, if firms can costlessly engage in reselling. Then, firms will wish to set up subsidiaries in state 2 that simply purchase and resell goods. This will increase sales in state 2 but have no effect otherwise, so it will lower taxes. Note that selling internally would not be recognized, but two firms could purchase each other's output in state 2 to accomplish the same thing.

Note, also, that in this model, a simple fix would be to exclude intermediate sales from the calculation, that is, allocate national value added based on retail sales alone. Then the optimization would be identical to that under a local tax on sales or value added. But, again, firms could avoid this simply by setting up separate resale companies with no value added, so that companies with value added would have no retail sales in state 1, and all retail sales would be handled by the dummy retail companies.

## Capital income tax with formula apportionment

Since the current system relies on three factors, assets, payroll, and sales, with sales given double weight, let us consider the effect of each of these factors, starting with sales weighting.

**Sales**

When capital income is apportioned using sales, the firm's objective is to maximize:

$$\left(p_1C_1 + p_2C_2\right) - \frac{r}{\left(1-\tau\dfrac{p_1\left(C_1+M_1\right)}{p_1\left(C_1+M_1\right)+p_2\left(C_2+M_2\right)}\right)}(K_1+K_2) - w_1L_1 - w_2L_2$$

where, as in the BNRT case, we know that output prices will not vary within a given state depending on whether the sales are final or intermediate. If we multiply through by the denominator with the tax term, we get, again using the definition of the apportionment term S in expression (6),

$$\frac{1}{(1-\tau S)}\left\{(1-\tau S)\left(p_1C_1 + p_2C_2 - w_1L_1 - w_2L_2\right) - r(K_1+K_2)\right\}$$

Because it will turn out that there are zero profits in equilibrium (because of constant returns to scale), maximizing the entire term will lead to the same outcome as maximizing the term in braces, so we can treat the term in braces as the maximand. This will simplify the problem, since then the analysis of the BNRT case can easily be adapted. Doing so, we set up the problem as before using the equality of production and consumption to substitute for $C_2$ and maximize with respect to the six factors and $C_1$. From the condition for $C_1$, we get:

(17) $$p_1 = \frac{p_2\left(1-(1-R')\tau S\right)}{1-R'\tau-(1-R')\tau S}$$

where

(18) $$R' = \frac{p_1C_1 + p_2C_2 - w_1L_1 + w_2L_2}{p_1\left(C_1+M_1\right)+p_2\left(C_2+M_2\right)}$$

is the ratio of corporate profits to sales in the two states together. Comparing expressions (17) – (18) to (15) – (16), the corresponding expressions for the BNRT, we can see that $R' < R$—the ratio of corporate profits to total sales is less than the ratio of retail sales to total sales, since the former deducts wages from the numerator.

The first-order conditions for capital in the two states are ($K_1$; same for $K_2$ except substitute $g$ for $f$):

$p_2f_1(1 - \tau S) + R'\tau S p_2 f_1 = r$

$\Rightarrow p_2f_1(1 - (1 - R')\tau S) = r$

Thus, we can again think of there being a general increase in prices consumption in both states—in this case $(1 - R')\tau S$—and an additional increase in prices of $R'\tau$ in state 1. Since $R' < R$, we can see that the general tax is bigger and the local tax smaller than in the case of the BNRT. However, this is true for a given tax rate, and the base of the capital income tax is smaller to the same extent.

As to labor, we have first-order conditions ($L_1$; same for $L_2$ except substitute $g$ for $f$):

$p_2f_2(1 - (1 - R')\tau S) = w_1(1 - \tau S)$

That is, there is a subsidy to labor offsetting the general output tax. For labor in state 2, this amounts to a net subsidy $\frac{R'\tau S}{1-\tau S}$. For state 1, using the expression for $p_1$, we get $p_2f_2(1 - R'\tau - (1 - R')\tau S) = w_1(1 - \tau S)$, or a net tax on labor (or capital owned by state 1 consumers) equal to $\frac{R'\tau(1-S)}{1-\tau S}$.

Finally, as to intermediate inputs, we have:

$M_1$: $$f_3 = \frac{1-\tau S}{1-R'\tau-(1-R')\tau S} = \frac{1-\tau S}{1-\tau S-R'\tau(1-S)} = 1 + \frac{R'\tau(1-S)}{1-R'\tau-(1-R')\tau S}$$

$M_2$: $$g_3 = \frac{1-\tau S}{1-(1-R')\tau S} = 1 - \frac{R'\tau S}{1-(1-R')\tau S}$$

Thus, compared to the BNRT, the capital income tax with the same apportionment factor is better for labor overall. However, it is important to emphasize that this model does not include distinct corporate and noncorporate sectors, so there is no distortion associated with taxing the two sectors differently as there is under existing state corporate tax systems. Alternatively, this analysis would apply in the case of distinct production sectors to an apportioned tax on all business income, not just corporate income.

The comparative statics analysis follows very closely that of the BNRT and so will not be laid out separately.

Finally, note that this tax system has one problem in common with the BNRT—firms have an incentive to increase turnover in state 2 via purchases and sales of goods there to reduce the share of their sales occurring in state 1.[1]

### Assets

The firm's objective when capital income is apportioned based on assets is to maximize:

$$\left(p_1C_1 + p_2C_2\right) - \frac{r}{\left(1-\tau\dfrac{K_1}{K_1+K_2}\right)}(K_1+K_2) - w_1L_1 - w_2L_2$$

As before, we multiply through by the denominator with the tax term to get the equivalent maximand,

$$\left\{(1-\tau T)\left(p_1C_1 + p_2C_2 - w_1L_1 - w_2L_2\right) - r(K_1+K_2)\right\}$$

where

$$T = \frac{K_1}{K_1+K_2}$$

Again, we set up the problem using the equality of production and consumption to substitute for $C_2$ and maximize with respect to the six factors and $C_1$. The conditions for $C_1$, intermediate inputs, and labor, are all very simple: $p_1 = p_2$ ; $f_3 = 1$ ; $g_3 = 1$ ; $p_2 f_3 = w_1$; $p_2 g_3 = w_2$ . For capital, the marginal conditions (using the zero-profits condition to substitute) are:

$$(1-\tau T)p_2 f_1 = r\left(1+\tau\frac{K_2}{K_1(1-\tau)+K_2}\right);\ (1-\tau T)p_2 g_1 = r\left(1-\tau\frac{K_1}{K_1(1-\tau)+K_2}\right)$$

from which it follows by taking a weighted sum that capital bears 100 percent of the tax, with capital facing an extra tax in state 1 and an offsetting subsidy in state 2. The shift of capital from state 1 to state 2 (assuming complementarity) will hurt labor in state 1 and help it in state 2. The comparative statics are straightforward.

### Payroll

When apportionment is based on payroll, the firm's objective is to maximize:

$$\left(p_1C_1 + p_2C_2\right) - \frac{r}{\left(1-\tau\dfrac{w_1L_1}{w_1L_1+w_2L_2}\right)}(K_1+K_2) - w_1L_1 - w_2L_2$$

As before, we multiply through by the denominator with the tax term to get the equivalent maximand,

$$\left\{(1-\tau Z)\left(p_1C_1 + p_2C_2 - w_1L_1 - w_2L_2\right) - r(K_1+K_2)\right\}$$

where

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[1] Such incentives under a corporate income tax with formula apportionment based on sales, for companies operating in high-tax states to increase sales in low tax states, have been recognized for some time. See Roger Gordon and John D. Wilson, "An Examination of Multijurisdictional Corporate Income Taxation under Formula Apportionment," *Econometrica* 54, no. 6 (1986): 1357–73.

$$Z = \frac{w_1 L_1}{w_1 L_1 + w_2 L_2}$$

Again, we set up the problem as before using the equality of production and consumption to substitute for $C_2$ and maximize with respect to the six factors and $C_1$. The conditions for $C_1$ and intermediate inputs are the same as for the case of asset apportionment, $p_1 = p_2$ ; $f_3 = 1$ ; $g_3 = 1$ .

For capital, the marginal conditions are also very simple in this case, because capital does not affect the share of income apportioned to state 1:

$$(1-\tau Z)p_2 f_1 = r \; ; \; (1-\tau Z)p_2 g_1 = r$$

It is evident from these conditions that, again, capital bears 100 percent of the tax. However, in this case there is no incentive to shift capital and no distortion of production and therefore no economic distortions. But this does not mean that labor will be unaffected. For labor, the conditions are:

$$(1-\tau Z)p_2 f_2 = w_1\left((1-\tau Z) + \frac{\tau(1-Z)}{(1-\tau Z)}\frac{rK}{w_1 L_1 + w_2 L_2}\right); (1-\tau Z)p_2 g_2 = w_2\left((1-\tau Z) - \frac{\tau Z}{(1-\tau Z)}\frac{rK}{w_1 L_1 + w_2 L_2}\right)$$

from which it follows that labor as a whole bears none of the tax (which is implied by the 100 burden on capital) but also that labor in state 1 loses and labor in state 2 gains.

## Illustrative Incidence Calculations

The incidence of these tax systems depends on the structure of production in California and elsewhere, as well as on the ability of firms to engage in the types of strategic avoidance behavior discussed for taxes based on formula apportionment using sales. However, to provide some idea of how the systems compare, we consider a concrete example in which these avoidance strategies are not used. We also assume a simple Cobb-Douglas structure for the production function,

$$X = AK^{\alpha_1} L^{\alpha_2} M^{\alpha_3}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$, and that the residents of state 1 have the same share of overall capital income as they do of labor income.

With these assumptions, the share of taxes borne by owners of labor and capital in states 1 and 2 under each of the tax systems discussed above are:

**TABLE A1**
**Incidence, Cobb-Douglas formulas**

| | Tax system | | | | | |
|---|---|---|---|---|---|---|
| **Factor** | **Final sales** | **Intermediate sales** | **BNRT** | **Capital income (assets only)** | **Capital income (payroll only)** | **Capital income (sales only)** |
| **Capital, state 1** | $\frac{\alpha_1}{\alpha_1+\alpha_2}$ | $S\frac{\alpha_1}{\alpha_1+\alpha_2}$ | $S\frac{\alpha_1\alpha_3}{\alpha_1+\alpha_2}+\alpha_1$ | $S$ | $S$ | $S(\alpha_2+\alpha_3)+\alpha_1$ |
| **Labor, state 1** | $\frac{\alpha_2}{\alpha_1+\alpha_2}$ | $(1+S)\frac{\alpha_2}{\alpha_1+\alpha_2}$ | $\alpha_3\left(1-S\frac{\alpha_1}{\alpha_1+\alpha_2}\right)+\alpha_2$ | $(1-S)$ | $(1-S)$ | $(1-S)(\alpha_2+\alpha_3)$ |
| **Capital, state 2** | 0 | $(1-S)\frac{\alpha_1}{\alpha_1+\alpha_2}$ | $(1-S)\frac{\alpha_1\alpha_3}{\alpha_1+\alpha_2}$ | $(1-S)$ | $(1-S)$ | $(1-S)(\alpha_2+\alpha_3)$ |
| **Labor, state 2** | 0 | $-S\frac{\alpha_2}{\alpha_1+\alpha_2}$ | $-(1-S)\frac{\alpha_1\alpha_3}{\alpha_1+\alpha_2}$ | $-(1-S)$ | $-(1-S)$ | $-(1-S)(\alpha_2+\alpha_3)$ |

where $S$ is the fraction of the economy accounted for by state 1. Because the incidence effects of small taxes are additive, we can combine the results from the first two columns using a weighted average to determine the incidence of a sales tax on final and intermediate sales (based on the share of each in total sales) and combine the results in the last three columns to determine the incidence of a capital income tax based on a three-factor formula, using the formula weights.

Using data from the Bureau of Economic Analysis for the United States in 2009, we choose the following parameter values[2]:

$\alpha_1$ = .1731

$\alpha_2$ = .3974

$\alpha_3$ = .4295

$S$ = .1337

$R$ = .5705

where $R$ is the ratio of final sales to total sales.

Applying these values to the expressions in Table A1 yields the following incidence estimates:

**TABLE A2**
**Incidence for specific parameter assumptions**

| | Tax system | | | | |
|---|---|---|---|---|---|
| **Factor** | **Final sales** | **Total sales (GRT)** | **BNRT** | **Capital income (sales 2x weighted)** | **Capital income (sales only)** |
| **Capital, state 1** | 0.303 | 0.190 | 0.190 | 0.209 | 0.284 |
| **Labor, state 1** | 0.697 | 0.737 | 0.810 | 0.791 | 0.716 |
| **Capital, state 2** | 0.000 | 0.113 | 0.113 | 0.791 | 0.716 |
| **Labor, state 2** | 0.000 | –0.040 | –0.113 | –0.791 | –0.716 |

Observe first that, for all but the tax on all sales, there is no net tax exporting—100 percent of the tax is borne by residents of state 1. As is evident from the expressions in Table A1, this lack of tax exporting is independent of the particular parameter values chosen; only the tax on intermediate sales succeeds in shifting some taxes out of state, because, unlike the apportioned taxes, it does not provide an implicit subsidy to any factors of production in state 2.

Beyond the issue of whether any of the tax is shifted to state 2 overall, however, there are net effects on out-of-state capital and labor separately, except for the case of the tax on final sales (as discussed above). In all other cases, out-of-state labor actually gains, at the expense of out-of-state capital owners, whose returns are depressed by the flight of capital from state 1. Capital's burden overall is highest under the two forms of capital income tax, equal to 100 percent of the total burden in both cases, but there is also a large subsidy to

[2] These parameter values are calculated as follows: $\alpha_3 = 1 - R$, the ratio of gross domestic product (GDP) to total output (from the 2009 input-output table). The ratio, $\alpha_1/\alpha_2$, equals the ratio of compensation of employees to all other national income, excluding proprietors' income. The share, $S$, equals the ratio of California GDP to U.S. GDP.

out-of-state labor in these cases. Still, in-state labor does better under either apportioned capital income tax than under the BNRT, but it does still better under the final sales tax, because there is no effective tax on intermediate inputs to drive capital away and therefore to reduce the productivity of labor.

Again, it is important to stress that these estimates are intended only for illustrative purposes. They depend on particular assumptions about the production function and about companies not using potential avoidance strategies under apportionment. Further, the calculations apply only for the introduction of small taxes, for which the tax burden approximately equals the revenue collected. That is, there are no deadweight costs of economic distortions included in these calculations. Since distortions vary among the tax systems, calculations taking these distortions into account would raise total burdens and could change the ranking of burdens across tax systems for any particular group. To get a sense of the importance of these distortions, we next consider the relative deadweight costs of the different systems.

## Illustrative Efficiency Calculations

As already mentioned, there are no first-order distortions associated with the introduction of small taxes, so we take the standard approach of using a second-order Taylor approximation to calculate deadweight loss.[3] We measure deadweight loss as the decline in total consumption, for which the second-order approximation is:

$$(19) \qquad -\frac{1}{2}\left[f_{11}\left(\frac{1}{S}\right)(dK_1)^2 + 2f_{13}(dK_1)\left(dM_1 - \frac{S}{1-S}dM_2\right) + f_{33}\left((dM_1)^2 + \frac{S}{1-S}(dM_2)^2\right)\right]$$

where expression (19) incorporates the fact that $dK_2 = -dK_1$. From this general expression, we can again derive formulas for each of the basic tax systems analyzed above under the assumption of Cobb-Douglas production. The results, expressed as a share of revenue, are shown in Table A3.

**TABLE A3**
**Deadweight loss as a share of revenue, Cobb-Douglas formulas**

| Tax system | Deadweight loss |
|---|---|
| **Final sales** | 0 |
| **Intermediate sales** | $\frac{1}{2}\left[\frac{1}{\alpha_1+\alpha_2}\left((1-S)\frac{\alpha_1}{\alpha_2}+\frac{1}{\alpha_3}\right)RG\right]$ |
| **BNRT** | $\frac{1}{2}\left[\frac{(1-S)}{\alpha_1+\alpha_2}\left(\frac{\alpha_1}{\alpha_2}+\frac{1}{\alpha_3}\right)R(1-R)G\right]$ |
| **Capital income (assets only)** | $\frac{1}{2}\left[(1-S)\frac{\alpha_1+\alpha_2}{\alpha_1\alpha_2}RG\right]$ |
| **Capital income (payroll only)** | 0 |
| **Capital income (sales only)** | $\frac{1}{2}\left[\frac{(1-S)}{\alpha_1+\alpha_2}\left(\frac{\alpha_1}{\alpha_2}+\frac{1}{\alpha_3}\right)R(1-R)G\right]$ |

where $G$ is the share of state 1 income collected as taxes. As already discussed, there are no distortions in this model under a tax on final sales or a capital income tax apportioned according to payroll. For the other tax systems, deadweight loss as a share of revenue is proportional to the share of income collected, consistent with

[3] See Alan J. Auerbach, "The Theory of Excess Burden and Optimal Taxation," in *Handbook of Public Economics* 1, ed. A. Auerbach and M. Feldstein (Amsterdam: North-Holland, 1985), pp. 61–127.

the standard result that deadweight loss rises in proportion to the *square* of the tax rate. The expression is the same for the two taxes apportioned by sales (the BNRT and the sales-weighted capital income tax) and varies in two respects for the tax on intermediate sales. One difference (the absence of the term $(1 - R)$) reflects the fact that the tax is only on intermediate sales, not total sales; this term difference would vanish for a tax on total sales, only part of which would apply to intermediate sales. The other difference (the absence of the term $(1 - S)$ multiplying the second term in parentheses) reflects the fact that, as discussed above, under the sales-apportioned taxes, the size of the distortion inducing a shift of intermediate production from state 1 to state 2 depends on the size of state 2.

For reasonable parameter values, the most distortionary of the taxes in Table A3 will be the capital income tax apportioned based on assets, because its denominator has the term $\alpha_1$, rather than $\alpha_1 + \alpha_2$. Intuitively, this difference results because, unlike the other distortionary taxes, this is the only tax that focuses entirely on capital, the factor that is mobile across states.

As with the incidence formulas applied in Table A2, we can combine these results to estimate the deadweight losses for particular tax systems of interest for particular parameter values.[4] Table A4 provides these estimates, where we set the parameter $G$ to .0534, the ratio of California state tax revenue to California GDP in 2009, with the revenue measure taken from the Census of Governments.

**TABLE A4**
**Deadweight loss as a share of revenue for specific parameter assumptions**

| | Tax system | | | | |
|---|---|---|---|---|---|
| | Final sales | Total sales (GRT) | BNRT | Capital income (sales 2x weighted) | Capital income (sales only) |
| **Deadweight loss (%)** | 0.00 | 1.22 | 1.18 | 2.89 | 1.18 |

These estimates may seem small, but one should keep in mind that they do not take into account the distortions associated with incomplete coverage of the taxes, other tax avoidance behavior, or interactions with federal taxes, whose existence would make the marginal effects of additional taxes at the state level *much* larger, because they would exacerbate existing distortions. However, their relative values are informative, showing that the capital income tax with sales double-weighted is the most distortionary, because of its inclusion of assets in its weighting scheme. The BNRT, sales-weighted capital income tax, and tax on total sales have similar distortions. Even though the tax on all sales (the GRT) is more distortionary than the other two taxes, the difference is not large, because California is small relative to the country as a whole. As discussed above, the GRT's overall distortion grows with the size of the adopting state relative to that imposed by the BNRT. Consequently, the efficiency difference between these two taxes would be much larger if we were evaluating the adoption of any particular tax in many or all states at the same time. The tax on final sales, as already discussed, causes no deadweight loss in this model.

Including variable labor supply would raise the deadweight loss associated with all the systems, including the tax on final sales. Another element from which the model abstracts is consumer choice among commodities. As discussed in the text, with the possibility of consumer choice there would be distortions induced by the

[4] In particular, the deadweight loss for the total sales tax is that for that portion of the revenue accounted for by the tax on intermediate sales, since the tax on final sales has no deadweight loss. For the case of formula apportionment, we simply use a weighted average of the three individual terms assuming that each raises all the revenue, which involves a small approximation error.

taxation of intermediate sales, so this omission understates the efficiency benefits of a tax on final sales relative to the other tax systems in Table A4. We also ignore transportation costs and imperfect substitutability of products of different states, both of which would tend to reduce behavioral responses to taxation and hence the inefficiencies of the various tax systems. Finally, because this is a static model with the overall supply of capital fixed, there is no potential for taxes to distort capital accumulation. Making the supply of capital sensitive to taxation would again increase the advantages of the tax on final sales relative to the other tax systems, all of which have some effect on the rate of return to capital. This extension would also increase the appeal of a shift away from asset-based apportionment for the capital income tax.

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PUBLIC POLICY INSTITUTE OF CALIFORNIA
500 Washington Street, Suite 600
San Francisco, California 94111
phone: 415.291.4400
fax: 415.291.4401
www.ppic.org

PPIC SACRAMENTO CENTER
Senator Office Building
1121 L Street, Suite 801
Sacramento, California 95814
phone: 916.440.1120
fax: 916.440.1121